As follows from (10), with an increase in the thickness of the deposition the diffusion flow to the rough surface diminishes, and this may lead to a decrease in the concentration of particles at the surface and to a reduction in the migration component of the rate of deposition.

## NOTATION

$a$, particle dimension; d, tube diameter; D, diffusion factor; C, particle concentration; g, acceleration of free fall; $j_D$, diffusion flow at the surface; R, $R_1$, inside and outside radii of the tube; $U_*$, dynamic velocity; v, particle volume; y, coordinate; $\alpha_1$, $\alpha_2$, coefficients of heat transfer from the internal and external media; $\beta = 1 - \delta/R$, choking factor; $\Delta$, roughness height; $\delta$, thickness of deposition; $\varepsilon_{re}$, resistance; $\varepsilon_R$, specific dissipation energy; $\nu$, kinematic viscosity; $\Delta\rho = \rho_{re} - \rho$; $\rho_{re}$, $\rho$, density of particles and the carrier phase; $\eta$, dynamic viscosity; $\tau$, stay time; $\tau_r$, relaxation time; $\tau_w$, tangential friction; $\varphi$, volumetric particle fraction. Subscript: 0, for a clean tube.

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# CHANGES IN THE STRUCTURE OF TURBULENT FLOWS SUBJECTED TO THE ACTION OF FLOW ACCELERATION


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UDC 532.517.4



*We examine the effects resulting from the laminarization of turbulent flows subjected to the action of flow acceleration. We describe the factors and conditions for the appearance of this phenomenon. As a theoretical base for this investigation we employ a mathematical model of a boundary layer for a broad range of turbulent Reynolds numbers, based on a modified e-ε turbulence model.*


The theory of hydrodynamic stability [1] rejects the possibility of a reverse transition from turbulent flow to laminar. However, in a number of experimental studies into turbulent flows [2, 3] a significant deviation was noted in the integral characteristics of heat exchange and friction, as well as in the profiles of the average velocity and temperature from those universal relationships applicable to a turbulent flow regime in the direction of relationships that are more in line with the laminar regime. This phenomenon has been designated as the laminarization of turbulent flows.

In their effort to generalize and systematize questions related to the phenomenon of turbulent-flow laminarization, the authors of [4], on the basis of studies that they carried out, came to the conclusion that it is possible to isolate certain external factors which, under these conditions, lead to a change in the mechanism of turbulent exchange:

flow acceleration which strives to reduce the extent to which the turbulent frictional stresses affect the average flow characteristics [3, 5];

the curvature of the streamlined surface, resulting in transverse flows through the channel [6];

the cooling of the boundary layer, which results in a tendency to stabilize the vortex structure of the boundary layer [7].


Moscow Automobile Construction Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 59, No. 2, pp. 196-202, August, 1990. Original article submitted July 7, 1989.





0022-0841/90/5902-0964$12.50

It is obvious that it is impossible to draw a clearly delineated boundary between the influence of these factors on the structure of turbulent flows, since they are interrelated with one another. For example, any increase in the curvature of the streamlined profile or an increase in the nonuniformity of the temperature field along and across the flow will lead to an increase in flow acceleration. Moreover, in real designs these factors may make themselves felt simultaneously. Therefore, the division of external factors affecting the laminarization of turbulent flows is conditional in nature, but useful from a methodological standpoint, enhancing the detailed study of individual aspects of the phenomenon.

In experimental studies into the flow of a gas in tubes with heated walls [8] note was also taken of the phenomenon of laminarization. It may be assumed that as the temperature rises there is an increase in the viscosity of the gas and, consequently, turbulent vortices are subjected to increasingly viscous damping as the gas makes its way through the tube. In this case, any turbulent gas flow will finally change over into laminar flow, provided that the walls are sufficiently heated over an adequate length. This may indeed be valid; however, it is obvious that just as in the case of boundary-layer cooling, we are dealing here with a nonuniformity in the temperature field, resulting in increased acceleration of the flow.

Bearing in mind the above-presented data, there is some point to examining in greater detail the conditions under which flow acceleration influences the structure of turbulent flows. Although the explanation for the laminarization of turbulent flows demands profound study into the very nature of the formation and disruption of turbulence, some of the quantitative relationships for this process can be predicted, relying on the integral momentum equation for the case of a plane stationary boundary layer within a compressible fluid, in the absence of any body-force effect [9]:

$$\frac{d\delta_2}{dx} + \frac{\delta_2}{U_\infty}\frac{dU_\infty}{dx}\left(2 + \frac{\delta_1}{\delta_2} - \mathrm{M}^2\right) = \frac{\tau_w}{\rho U_\infty^2}. \quad (1)$$

If the velocity profile in the turbulent boundary layer corresponds to the 1/7 rule, the formula for the local coefficient of friction can be obtained in the form

$$\frac{\tau_w}{\rho U_\infty^2} = 0{,}0128\,(U_\infty \delta_2/\nu)^{-1/4}. \quad (2)$$

Having substituted into (1) the value of $\tau_w$ from (2) and the ratio of the conditional boundary-layer thicknesses, equal to 1.29, after introduction of the momentum-loss thickness into the Reynolds number we derive the following equation:

$$\frac{\nu}{(2{,}29 - \mathrm{M}^2)\,U_\infty \mathrm{Re}_{\delta_2}}\,\frac{d\,\mathrm{Re}_{\delta_2}}{dx} = \frac{0{,}0128}{(2{,}29 - \mathrm{M}^2)\,\mathrm{Re}_{\delta_2}^{1,25}} - \left(\frac{\nu}{U_\infty^2}\frac{dU_\infty}{dx}\right). \quad (3)$$

With large accelerations of the flow, the last of the terms in Eq. (3) increases. The gradient in the Reynolds number may then become negative, which leads to a reduction in $\mathrm{Re}_{\delta_2}$. If we draw the entirely logical, even if quite coarse, conclusion to the effect that laminarization of turbulent flows begins at the same Reynolds numbers as in the direct transition [$(\mathrm{Re}_{\delta_2})_w = 360$], in the case of flows with $\mathrm{M} \ll 1$ we can obtain the dimensionless acceleration parameter which defines the onset of laminarization:

$$K = \frac{\nu}{U_\infty^2}\frac{dU_\infty}{dx} > 3{,}5 \cdot 10^{-6}. \quad (4)$$

It has been established experimentally [2, 4, 5] that a reduction in the characteristics of heat exchange begins to make itself evident in flows with $K \sim 2\text{-}4 \cdot 10^{-6}$. However, this criterion does not allow for a quantitative evaluation of the effects of laminarization in turbulent flows.

Experimental and theoretical data [3, 10] have demonstrated that the so-called reverse transition of the turbulent flow regime into a laminar regime under the action of flow acceleration. For any levels of acceleration, even with $K > 10^{-5}$, a high level of pulsations in velocity and temperature is maintained, i.e., the flow cannot be regarded as laminar. It is assumed that laminarization is a special specific form of the turbulent flow regime and physical and mathematical models of turbulent transport may serve as its theoretical basis.

Theoretical studies of turbulent boundary layers in accelerating flows [10] have demonstrated the possibility of describing the effects of laminarization by means of a mathematical model of a boundary layer for a broad range of turbulent Reynolds numbers [11], based on the modified two-parameter e–ε turbulence model. The utilization of this model allows in greater detail to examine the processes of heat and momentum transfer in accelerated flows, to determine the features and conditions for the phenomena of laminarization, and to describe the nature of the changes occurring within the structure of turbulent flows, ascribed to the effect of a negative pressure gradient. Moreover, the carrying out of a numerical

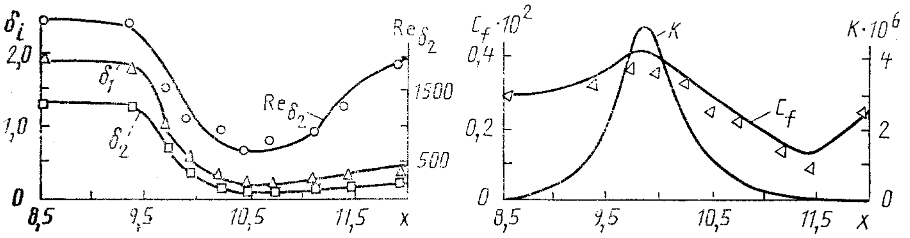


Fig. 1. Distribution of conditional boundary-layer thicknesses (expulsion $\delta_1$ and loss of momentum $\delta_2$), of the Reynolds number $Re_{\delta_2}$, and of the coefficient of friction $C_f$ in the effective zone of flow acceleration. Points represent experimental data [3]; curves represent calculations based on the model. $\delta_i$, x, m.

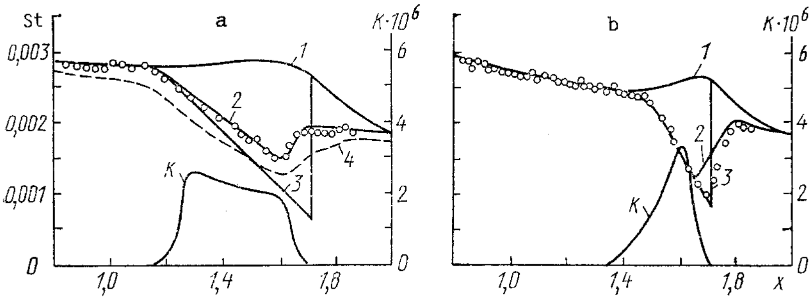


Fig. 2. Calculation of the heat exchange in the effective flow-acceleration zone: a) moderate flow acceleration ($K_{max} = 2.52 \cdot 10^{-6}$; b) strong flow acceleration ($K_{max} = 3.39 \cdot 10^{-6}$). Points represent experimental data [2]; 1) calculations based on the model [13]; 2) calculation based on the proposed model; 3) calculations based on the model [13], but with K > 0 $- \rho \langle u'v' \rangle$ = const; 4) calculation based on the model [14].

experiment on the basis of this model will arm the researcher with new data that have not become available through empirical means up to the present time.

In accordance with [11], considering the equations of energy for the determination of the transfer of heat, the mathematical model for the boundary layer in the case of a broad range of turbulent Reynolds numbers include:

the system of equations

$$\frac{\partial \rho U}{\partial x} + \frac{\partial \rho V}{\partial y} = 0, \quad \rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial U}{\partial y}\right] - \frac{dp}{dx},$$

$$C_p \rho U \frac{\partial T}{\partial x} + C_p \rho V \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[(\lambda + \lambda_t)\frac{\partial T}{\partial y}\right] + U\frac{dp}{dx} + \mu\left(\frac{\partial U}{\partial y}\right)^2 + \rho\varepsilon,$$

$$\rho U \frac{\partial e}{\partial x} + \rho V \frac{\partial e}{\partial y} = \frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial e}{\partial y}\right] + \mu_t\left(\frac{\partial U}{\partial y}\right)^2 - \rho\varepsilon, \tag{5}$$

$$\rho U \frac{\partial \varepsilon}{\partial x} + \rho V \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] + C_1 \mu_t \frac{\varepsilon}{e}\left(\frac{\partial U}{\partial y}\right)^2 - C_2 f \frac{\rho \varepsilon^2}{e};$$

the closing relationships

$$\mu_t = C_\mu \mu R_t, \quad \lambda_t = \frac{C_p \mu_t}{Pr_t}, \quad Pr_t = 0{,}9, \quad \sigma_\varepsilon = 1{,}3, \quad C_1 = 1{,}65, \quad C_2 = 2\,[1 - 0{,}3\exp(-R_t^2)],$$

$$C_\mu = 0{,}095\,[-\exp(-2{,}5) + \exp(-125/(50 + R_t))], \quad f = -\exp(-10) + \exp(-250/(25 + y_*^3)); \tag{6}$$

the boundary conditions

$$y = 0 \quad U_w = V_w = e_w = \varepsilon_w = 0, \quad T_w = \varphi(x),$$

$$y \to \infty \quad \frac{\partial U}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial e}{\partial y} = \frac{\partial \varepsilon}{\partial y} = 0. \tag{7}$$

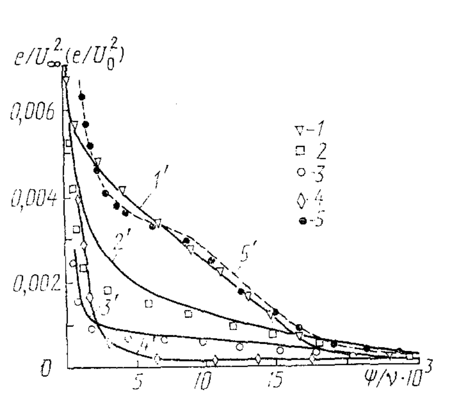


Fig. 3

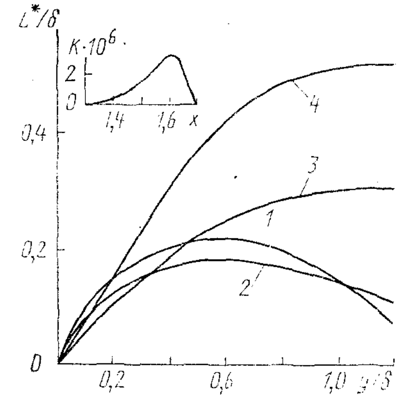


Fig. 4

Fig. 3. Changes in the intensity of turbulence in the effective flow-acceleration zone: points represent experimental data [3]; curves represent calculations based on the models; 1, 1′) $e/U_\infty^2$ and $e/U_0^2$ for x = 9.36 m; 2, 2′) $e/U_\infty^2$ for x = 9.88 m; 3, 3′) $e/U_\infty^2$ for x = 10.16 m; 4, 4′) $e/U_\infty^2$ for x = 10.77 m; 5, 5′) $e/U_0^2$ for x = 10.16 m; the acceleration factor K corresponds to the values of x shown in Fig. 1.

Fig. 4. Distribution of the integral turbulence scale in the effective zone of flow acceleration: 1) 1.3 m; 2) 1.45; 3) 1.58; 4) 1.67.

Equations (5) in conjunction with conditions (6) and (7), as well as in conjunction with the equation of state and the relationships $\mu = \mu(p, T)$, $\lambda = \lambda(p, T)$, $C_p = C_p(p, T)$, and $R = R(p, T)$ were all solved on a computer. The initial equations and boundary conditions were approximated by finite-difference analogs written in an implicit scheme. The initial profiles were based on experimental and theoretical data for the laminar and turbulent flow regimes [9, 12]. The system of algebraic equations derived here was solved by the standard sweeping method. To speed up the calculation process and to ensure its accuracy, we introduced a modified system of coordinates to "compress" the lateral coordinate near the wall. Satisfactory calculational accuracy was achieved for nongradient flow regimes at 40 points across the calculation grid, and in the case of accelerated flows at 100 points.

Figure 1 shows the results obtained in a calculation based on the model as compared to the experimental data from [3]. As we can see from these curves, although the calculation yields somewhat elevated results, the qualitative coincidence with experiment is satisfactory. Acceleration of the flow in the interval of a positive gradient for the parameter K leads to a pronounced reduction in the thickness of the dynamic boundary layer ($\delta_1$ and $\delta_2$, the conditional thicknesses of expulsion and loss of momentum). This reduction is so significant that even despite the simultaneous increase in the velocity $U_\infty$ of the incident flow, we have a reduction in the Reynolds number $Re_{\delta_2}$. It should be noted that these parameters reach their minimum downstream of the point at which K is at its maximum. It has been established at the same time that the relative thickness of the viscous laminar sublayer $\delta_\ell$ increases under these conditions. The pronounced changes in the thickness of the dynamic boundary layer are accompanied by an increase in the local tangential frictional stress at the wall during the initial stage of flow acceleration. The reduction in the coefficient of friction $C_f$ begins in the region of negative values for the gradient of the parameter K. Here the minimum value of $C_f$ for x = 11.5 m is smaller by a factor of more than two than the values of the coefficient $C_f$ for a nongradient regime.

When the accelerating action (K = 0) comes to an end, the characteristics of the boundary layer approach quantities corresponding to developed turbulent flow. However, since the velocity $U_\infty$ of the external flow increased substantially, the conditional thicknesses of the boundary layer became smaller.

The possibilities for the proposed model in the description of heat exchange in turbulent boundary layers in the case of accelerated flows were evaluated through comparison against experimental data [2] (the points in Fig. 2). The velocity of the incident flow, the difference in temperature between the wall and the external flow, experimentally derived, were utilized in the calculation as the boundary conditions. We can see from the cited data that the action of the negative pressure gradients on the flow leads to a reduction in the intensity of heat exchange. In this case, not all of the models used for the calculation yield a correct description of the experiment. The nature of the change in the Stanton number, calculated on the basis of the method covered in [13] (curve 1), may erroneously lead to the conclusion that there exists a Reynolds analogy between the processes of heat and momentum transfer in the accelerated flow. A drawback of this model is the fact that it specifies the magnitude of the integral turbulence dimension L in the form of a functional parametric dependence exclusively on the dimensionless lateral coordinate. In complex flows involving injection, under nonisothermal conditions or in the flows being considered here, governed by the presence of a strong pressure gradient, the dimension L proves to be a function of a larger number of variables, as will be demonstrated below.

In the case of moderate flow accelerations ($K_{max} = 2.52 \cdot 10^{-6}$), acting at a considerable distance along the surface being studied (Fig. 2a), the best coincidence with experimental data is offered by the proposed model. Calculations based on the model in [14] lead to less exact results (curve 4).

As follows from Fig. 2b, when the acceleration is in the form of a brief spike, the calculational accuracy is diminished (curve 2), but the proposed model allows us correctly to evaluate the region of reduced heat-exchange intensity.

According to the qualitative scheme for developed turbulence [1], perturbations of various dimensions are simultaneously present in a turbulent flow. With transition to the more minor of the pulsations, in addition to the weakening of the orienting effect of the averaged flow on the flow exhibiting pulsations, the influence of all of its geometric and kinematic features is also weakened. It may be assumed that the characteristics of the averaged flow do not directly define the statistical regime of very small-scale pulsations. The averaged flow affects the very small-scale pulsations only indirectly in terms of the magnitude of that energy flux which is transferred from the flow through all of the perturbations of various magnitudes and is scattered finally, changing into heat. Thus, sharp changes in the parameters of the averaged flow will affect the pulsations of the flow only after a certain period of time has elapsed, which is the same as saying only after some distance downstream in the direction of the flow has been covered.

Analysis of the experimental data [3] and the results of the numerical study into the proposed boundary-layer model for a broad range of turbulent Reynolds numbers, such as those shown in Fig. 3, also provides a basis for the contention that some inertia exists in the parameters of the fluctuating transfer relative to changes in the averaged flow. The graphs show the distribution of the relative $e/U_\infty^2$ and absolute $e/U_0^2$ magnitudes of intensity in turbulence e in the direction of the streamline $\psi$ as a function of the acceleration ratio. The relative values of $e/U_\infty^2$ (curves 1′-4′) in the effective flow acceleration zone diminish i.e., we can speak of a reduction in the relative intensity of flow turbulence. This leads to a disruption of the conditions (generally accepted for the calculation of turbulent flows) under which the pulsating components affect the averaged parameters of the flow. At the same time, the absolute values of $e/U_0^2$ (curve 5′) remain virtually unchanged, at least in the initial stage of the acceleration.

It is entirely obvious that the pulsating motion exhibits a certain "memory" of the development of the flow, and this must be taken into consideration in calculating the accelerated flows. Models which fail to take this into consideration, such as, for example, [13], do not permit correct description of the effect of laminarization in turbulent flows.

In the present investigation we confirmed the fact of the existence of inertia in the parameters of fluctuating transfer and in its influence on the averaged characteristics of the flow, this having been accomplished by computation. We made use of the following computation algorithm. Before and beyond the effective zone of the negative pressure gradient the solution is achieved on the basis of one of the turbulent models, for example, the one based on the hypothesis of the length of the displacement path or the one which takes into consideration the equation for intensity of turbulence [13]. Over the extent of the entire flow acceleration region the magnitude of the turbulent Reynolds stress is "frozen in," i.e., it remains equal to $-\rho\langle u'v'\rangle_0$, which is the value of the Reynolds stress at the point at which the effect of the negative pressure gradient sets in.

The computational results obtained on the basis of this algorithm are shown in Fig. 2 (curve 3). Excellent coincidence between theoretical and experimental data is observed for the case of sharp spiking values of the acceleration parameter (Fig. 2b). However, since it is only the pressure gradient that again becomes equal to zero, on the basis of the proposed algorithm, the Reynolds stress ceases to be a constant quantity. In this case, the characteristics of heat exchange coincides with the calculation data, for example, based on the model from [13], but differ significantly from the experimental data.

Moderate pressure gradients (curve 3 in Fig. 2a) exert no such significant influence on the pulsation characteristics and the assumption of constancy in $-\rho\langle u'v'\rangle_0$ in the effective acceleration zone leads to erroneous results.

If we take into consideration that the mathematical model of the boundary layer for a broad range of turbulent Reynolds numbers [11] for all the cited data satisfactorily describes the effect of laminarization, it becomes possible on the basis of this model to achieve an estimate through some of the fluctuating flow characteristics not earlier achieved experimentally. For example, let us examine the changes in the integral dimension of turbulence in the zone of effective flow acceleration. Using the Rotta formula for dissipation of tubulent energy [12]:

$$\varepsilon = \left(\frac{3{,}93}{\mathrm{Re}_t} + 0{,}202\right)\frac{e^{3/2}}{L},$$

by processing the results from the calculation of the accelerated flows on the basis of the proposed boundary-layer model we can derive the values for L* (Fig. 4). The dimension L* is only an analog of the integral scale [12]. However, qualitatively they are similar and it might be asserted that the changes in L* are analogous to changes in L in the zone of effective flow acceleration.

## NOTATION

x, coordinate in the direction of the flow; y, coordinate transverse to the flow; U, velocity component along the x axis; V, velocity component along the y axis; p, pressure; T, temperature; u′, v′, w′, or $u_i'$, components of pulsating velocity; i = 1, 2, 3; $e = \frac{1}{2}\sum_{i=1}^{3}\langle(u_i')^2\rangle$ is the intensity of turbulence; $\varepsilon = \nu\sum_{i,j=1}^{3}\langle(\partial u_i'/\partial x_j)^2\rangle$, isotropic portion of total dissipation of turbulent energy; $\mu$, dynamic viscosity; $\nu$, kinematic viscosity; $\rho$, density; $\lambda$, coefficient of heat transfer; $C_p$, heat capacity of fluid at constant pressure; $-\rho\langle u'v'\rangle$, turbulent tangential frictional stress; $U_\infty$, velocity at boundary of boundary layer; $U_0$, velocity at boundary of boundary layer at the point at which flow acceleration begins; $\tau_w$, frictional stress on streamlined surface; $\delta$, boundary-layer thickness; $\delta_1$, $\delta_2$, conditional thicknesses of expulsion and loss of momentum; M, Mach number; $\mathrm{Re}_{\delta_2}$, Reynolds number calculated no the basis of the conditional thickness of loss of momentum; K, parameter of flow acceleration; $\mu_t$, coefficient of turbulent exchange; $\lambda_t$, coefficient of turbulent heat transfer; $\mathrm{Pr}_t = c_p\mu_t/\lambda_t$, turbulent Prandtl numbers; $\mathrm{Re}_t = \sqrt{e}L/\nu$, $R_t = e^2/(\nu\varepsilon)$, turbulent Reynolds numbers; $C_f$, local coefficient of friction; f, correction function in equation for dissipation; $C_1$, $C_2$, $C_\mu$, $\sigma_\varepsilon$, coefficients; $y_* = yU_\tau/\nu$, dimensionless coordinates across the flow; $U_\tau = \sqrt{\tau_w/\rho_w}$, dynamic velocity; R, gas constant; St, Stanton number; L, integral turbulence scale; L*, analog of integral turbulence scale; $\delta_l$, thickness of the laminar viscous sublayer; $\psi$, stream function.

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